GG-5210 Problem Set #2
Autumn Semester, 2000
Due October 06, 2000---Start Now!

Vector Operations And Stress Transformations

Do problems 2-1 to 2-5 <u>without</u> MatLab or Maple

2-1. a) Compute the direction cosine matrix for the transformation of the axes x_i to x_i' given the following rotation of axes $(x_1, x_1') = 20°$, $(x_2, x_2') = 0°$, and $(x_3, x_3') = 20°$. Assume the rotation is counterclockwise about the x_2-axis.

b) Transform the vector $V_O = (3,1,0)$ from the "old coordinates", x_i, to the "new coordinates", x_i'.

c) Calculate the magnitude of both vectors.

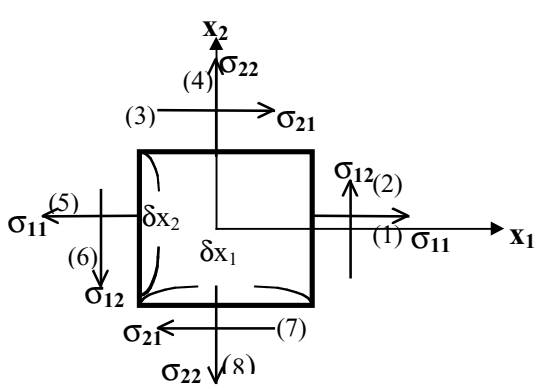
2-2. The stress tensor values at a point P are given by the matrix.

$$\sigma_{ij} = \begin{bmatrix} 8 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 7 \end{bmatrix}$$

Determine the normal and shear stresses on the plane whose unit normal vector at P is:

$$\vec{n} = (2/3)\hat{\mathbf{i}} - (2/3)\hat{\mathbf{j}} + (1/3)\hat{\mathbf{k}}$$

2-3 Consider an elemental area in static equilibrium, i.e, the sum of the torque s, is zero. The state of stress is shown below:



a) Sum the forces in the x_1 and x_2 direction and write the resulting equation using indicial notation. These are the equations of stress equilibrium for the two dimensional case.

b) Solve for the moments for a body in rotational equilibrium, and show that in general the stress tensor is symmetric, i.e. that $\sigma_{ij} = \sigma_{ji}$. Recall that the moment is the product of force and moment arm.

2-4 By visual inspection (do no multiplication or division)

$$A = \begin{bmatrix} 15 & 3 & 4 \\ 3 & 0 & \sqrt{29} \\ 4 & \sqrt{29} & -8 \end{bmatrix} \qquad B = \begin{bmatrix} 13 & 2 & 0 \\ -1 & 12 & 5 \\ 8 & -2 & \sqrt{13} \end{bmatrix}$$

Do you think tensor B could be derived from tensor A through an orthonormal change in coordinate axes? Why? or Why not? Give at least 2 reasons.

Do The Next Problems Using MatLab/Maple

Write scripts in MatLab (or Maple) for the following problems. Your routines should include comment lines explaining the purpose of the routine and the various inputs and outputs. If you have already done part of these problems in Problem Set 1 of course just use those programs.

2-5. For an input vector in three dimensions.

a) find the magnitude of a vector.
b) find the sum of two vectors.
c) find the scalar product of two vectors.
d) find the vector product of two vectors.

2-6. Write a script using the necessary routines from problem 1 to find the angle between two vectors.

2-7. Use your scripts developed in problems 1 and 2 to find the magnitude, sum, scalar product, vector product and angle between the vectors (1,4,2) and (2,3,1).

2-8. a) Write a script to multiply a 3x3 matrix by a vector (element by element).

b) Write a script to multiply a 3x3 matrix by another 3x3 matrix (element by element).

2-9 a) Write a subroutine that uses Gaussian elimination to solve the system of equations $A\mathbf{x} = \mathbf{b}$. Your routine should take an arbitrary 3x3 (we're making it easy!) matrix A and 3 element vector **b** as inputs.

b) Use it to solve for **x** in:

$$\begin{bmatrix} 10 & -7 & 0 \\ -3 & 2 & 6 \\ 5 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 6 \end{bmatrix}$$

c) Test your answer by working the problem backwards.

2-10 a) Use the MatLab INV routine to invert the matrix:

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

b) Test the solution by using an operation of the form $A^{-1}A$ using the scripts in problem 2-10.

2-11. a) Write a script to find the eigenvalues and eigenvectors of a real, symmetric 3x3 matrix.

b) Use this script to find the eigenvalues and eigenvectors of:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

c) Verify that the solution is correct by using a subroutine from the above problem to show that the eigenvectors and eigenvalues satisfy their definition.

2-12. Let:

$$\sigma_{ij} = \begin{bmatrix} 10 & 0 & 2 \\ 0 & 8 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

be the stress tensor at some point, P. Find the stress vector acting on a plane parallel to the plane 2x + y + 3z = 0 at that point.